Received: *May 14, 2000*, Accepted: *August 9, 2000*
Revised: *August 1, 2000*

Non-commutative gauge dynamics from the string worldsheet

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Abstract: We show how string theory can be used to reproduce the one-loop two-point photon amplitude in non-commutative U(1) gauge theory. Using a simple realization of the gauge theory in bosonic string theory, we extract from a string cylinder computation in the decoupling limit the exact one loop field theory result. The result is obtained entirely from the region of moduli space where massless open strings dominate. Our computation indicates that the unusual IR/UV singularities of non-commutative field theory do not come from closed string modes in any simple way.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Superstring Vacua.

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JHEP08(2000)011

Contents

1. Introduction

It is striking that field theories on non-commutative spaces are naturally embedded in string theory. The first complete example of this phenomenon was found in toroidal compactifications of Matrix theory with a non-zero B field [1, 2]. Of course the remarkable early construction of membranes by large matrices [3] is very much in this spirit. After [1, 2] additional work was done on extracting non-commutative Yang-Mills theory more directly from open string theory [4]–[8]. In a sense this work culminated in [9] where, among other things, a decoupling limit was carefully formulated in which perturbative open string theory reduced to non-commutative Yang-Mills theory. A large amount of work, partially inspired by these developments, has been done on the perturbative dynamics of non-commutative field theories [10]–[30]. These field theories have a very interesting and unusual perturbative behavior [26]–[30]. The noncommutativity of the underlying space gives rise to a strong mixing between the ultraviolet and the infrared [26, 27]. There are analogs of this IR/UV mixing in string theory which provides one motivation to study these systems. When loop diagrams are evaluated in these theories, large momentum regions of the loop integration lead to terms in the effective action that are infrared divergent and non analytic in the noncommutativity parameter θ. In conventional field theory, singularities in the low energy effective action usually reflect omission of relevant low energy degrees of freedom, and the low energy description is cured once the the missing degrees of freedom are added. In [26, 27], it was proposed that at least some of the novel IR/UV divergences of one loop diagrams in the non-commutative theory could be understood as arising from tree level exchange of new degrees of freedom. This phenomenon is analogous to open-closed channel duality in one loop string graphs, where ultraviolet divergences in the open string channel can be interpreted as infrared divergences arising from tree level exchange of massless closed strings. This work is motivated in part by trying to understand this interpretation of the IR/UV singularities in non-commutative field theories.

Non-commutative gauge theories can be realized in string theory by taking a low energy decoupling limit of theories on D-branes, in the presence of a constant magnetic field [1, 2, 9]. In this stringy setup one can try to reproduce the non-commutative perturbative expansion from string theory. This embedding confronts the issue of whether extra degrees of freedom — apart from the obvious massless open string modes — are needed to make sense of the low energy effective action. As we will show, there seems to be no need to add any further degrees of freedom. We can account for the entire field theory result by looking at the region of cylinder moduli space which is dominated by massless open strings. The opposite region of moduli space, where massless closed strings dominate, gives a vanishing contribution in the zero-slope limit and therefore seem to decouple in the field theory limit. This seems to indicate that the degrees of freedom proposed in [26, 27], even though they reproduce the low energy effective action after integrating them out, do not have a natural interpretation as massless closed string states.

In section 2 we compute the two-point function of the pure non-commutative U(1) gauge theory in $3+1$ dimensions at one loop in the background field gauge. The background field gauge is very useful for comparing field theory amplitudes with the zero slope limit of string amplitudes because the effective action obtained in the background field gauge is manifestly gauge invariant, as is the answer obtained in string theory. At one loop, the two point function of the non-commutative gauge theory has terms which contain logarithmic and quadratic [28, 30] infrared divergences which do not appear in conventional gauge theories. The appearance of quadratic infrared divergences is surprising, but nevertheless compatible with gauge invariance.

In section 3 we embed this gauge theory in bosonic string theory by considering the low energy limit of the theory on a single D3-brane stuck at an $\mathbb{R}^{22}/\mathbb{Z}_2$ orbifold singularity and in the presence of a magnetic field along the worldvolume directions.[31, 32][1] We compute at one loop the planar and non-planar contributions to the two-point function of photons in the string theory. The field theory answer is reproduced by isolating from the string theory amplitude the contribution coming from the boundary of cylinder moduli space where massless open strings are important. Our result indicates that closed string modes decouple from the low energy non-commutative field theory, just as they decouple from conventional gauge theories realized on branes. We conclude with a discussion of our results in section 4.

[1] In this paper we do not discuss the physics of non-commutative field theories with $\theta^{0i} \neq 0$. Such a gauge theory can be realized by having a constant electric field along the brane. However, in the decoupling limit the upper bound of the allowed electric field vanishes and the string theory realization is ill defined.

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2. Gauge theory calculation

In this section we describe the calculation of the two point function in non-commutative U(1) gauge theory in the background field gauge. The action for the U(1) non-commutative Yang-Mills theory in a background metric $G^{\mu\nu}$ is given by

$$S = -\frac{1}{4}\int d^4x\sqrt{-G}G^{\mu\rho}G^{\nu\sigma}F_{\mu\nu}\star F_{\rho\sigma}\,, \tag{2.1}$$

where the field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]\,, \qquad [A_\mu, A_\nu] = A_\mu \star A_\nu - A_\nu \star A_\mu\,. \tag{2.2}$$

The action in (2.1) is invariant under the gauge transformation

$$\delta_\lambda A_\mu = \partial_\mu \lambda - ig[A_\mu, \lambda] \equiv D_\mu \lambda\,. \tag{2.3}$$

The non-commutative star product appearing in (2.1) and (2.2) is defined by

$$f(x)\star g(x) = e^{\frac{i}{2}\theta^{ij}\frac{\partial}{\partial\alpha_i}\frac{\partial}{\partial\beta_j}} f(x+\alpha)\,g(x+\beta)\Big|_{\alpha=\beta=0}\,, \tag{2.4}$$

where the parameter θ^{ij} is related to the commutator of the coordinates in the non-commutative space:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}\,. \tag{2.5}$$

We will quantize the theory in background field gauge [33, 34]. The gauge field A_μ is split into classical and a quantum pieces denoted A_μ and Q_μ, respectively. The path integral is performed over the quantum fields while the classical fields are kept fixed. The generating functional for Green's functions is given by

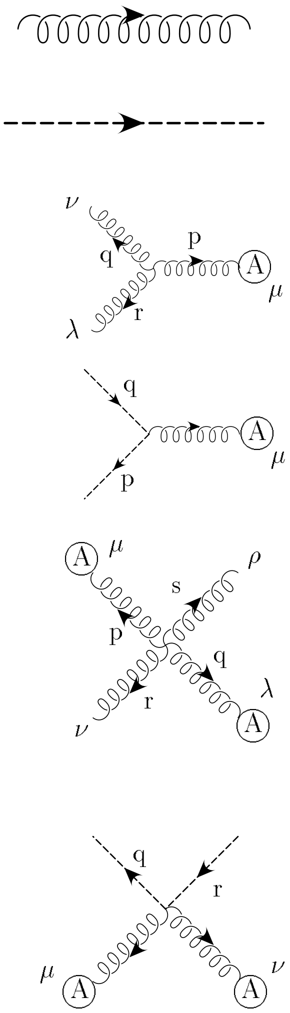
$$\begin{aligned} Z[J, A] = \int [dQ]\,\det\left[\frac{\delta\Delta}{\delta\lambda}\right]\exp\Bigg[i\int d^4x\sqrt{-G}\bigg(&-\frac{1}{4}G^{\mu\rho}G^{\nu\sigma}F_{\mu\nu}\star F_{\rho\sigma} - \\ &-\frac{1}{2\alpha}\Delta^2 + G^{\mu\nu}J_\mu Q_\nu\bigg)\Bigg]\,, \end{aligned} \tag{2.6}$$

where Δ is the gauge fixing condition, $\det[\delta\Delta/\delta\lambda]$ is the Faddeev-Popov determinant and J_μ is an external current coupled only to the quantum fields. In (2.6), $F_{\mu\nu}$ is understood to be a function of both A_μ and Q_μ. The background field gauge effective action is defined by

$$\Gamma[\bar{Q}, A] = W[J, A] - \int d^4x\sqrt{-G}G^{\mu\nu}J_\mu\bar{Q}_\nu\,, \tag{2.7}$$

where

$$W[J, A] = -i\,\ln\,Z[J, A]\,, \qquad \text{and} \qquad \bar{Q}_\mu = \frac{\delta W}{\delta J^\mu}\,. \tag{2.8}$$



$$i(-G_{\mu\nu} + (1-\alpha)\tfrac{p_\mu p_\nu}{p^2})/(p^2+i\epsilon)$$

$$i/(p^2+i\epsilon)$$

$$2g\left[G_{\mu\lambda}(p_\nu - r_\nu - \tfrac{1}{\alpha}q_\nu) + G_{\nu\lambda}(r_\mu - q_\mu) + G_{\mu\nu}(q_\lambda - p_\lambda + \tfrac{1}{\alpha}r_\lambda)\right]\sin(\tfrac{1}{2}p\tilde{q})$$

$$2g\sin(\tfrac{1}{2}p\tilde{q})\,(p_\mu + q_\nu)$$

$$\begin{aligned}- i4g^2 \Big[&\sin(\tfrac{1}{2}\tilde{p}r)\sin(\tfrac{1}{2}\tilde{q}s)(G_{\mu\lambda}G_{\nu\rho} - G_{\mu\rho}G_{\nu\lambda} + \tfrac{1}{\alpha}G_{\mu\nu}G_{\rho\lambda})\\ &+ \sin(\tfrac{1}{2}\tilde{p}s)\sin(\tfrac{1}{2}\tilde{r}q)(G_{\mu\nu}G_{\lambda\rho} - G_{\mu\lambda}G_{\nu\rho} - \tfrac{1}{\alpha}G_{\mu\rho}G_{\nu\lambda})\\ &+\sin(\tfrac{1}{2}\tilde{p}q)\sin(\tfrac{1}{2}\tilde{r}s)(G_{\mu\nu}G_{\lambda\rho} - G_{\mu\rho}G_{\nu\lambda})\Big]\end{aligned}$$

$$- i4g^2 G_{\mu\nu}\left[\sin(\tfrac{1}{2}\tilde{p}q)\sin(\tfrac{1}{2}\tilde{s}r) + \sin(\tfrac{1}{2}\tilde{s}q)\sin(\tfrac{1}{2}\tilde{p}r)\right]$$

Figure 1: Feynman rules for non-commutative U(1) in background field gauge. In the calculation of section 2, we use Feynman gauge, $\alpha = 1$.

The background field gauge effective action is invariant under the transformations

$$\delta A_\mu = \partial_\mu\lambda - ig[A_\mu, \lambda]\,, \qquad \delta\bar{Q}_\mu = [\lambda, \bar{Q}_\mu]\,, \tag{2.9}$$

so $\Gamma[0, A]$ is a manifestly gauge invariant functional of the classical field A_μ.

We can compute $\Gamma[0, A]$ by summing one-particle irreducible Feynman diagrams with classical fields A_μ on external legs and quantum fields Q_μ appearing only in internal lines. The Feynman rules for conventional non-abelian gauge theory in the background field gauge are given in [34]. The Feynman rules for the non-commutative U(1) theory can be obtained from [34] by simply replacing the structure constants f_{abc} by $\sin(\frac{1}{2}p_\mu\theta^{\mu\nu}k_\nu)$, where p, k are the momenta of two of the gluons entering the vertex. The Feynman rules relevant for the calculations of this paper are shown in figure 1.

The explicit gauge invariance of $\Gamma[0, A]$ simplifies computations in the gauge theory. The bare field strength, when expressed in terms of the renormalized coupling and gauge field, is

$$F_{\mu\nu}^{\text{bare}} = Z_A^{1/2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu - igZ_g Z_A^{1/2} [A_\mu, A_\nu] \right) , \tag{2.10}$$

where the Z_g, Z_A are the coupling constant and field renormalizations

$$g^{\text{bare}} = Z_g g \,, \qquad A_\mu^{\text{bare}} = Z_A^{1/2} A_\mu^{\text{bare}} \,. \tag{2.11}$$

Gauge invariance of (2.10) implies that $Z_A^{-1/2} = Z_g$. This means that the β-function can be computed from Z_A, which only requires knowledge of the two-point function of the theory.

The background field gauge is also very useful for comparing field theory amplitudes with the zero slope limit of string theory amplitudes. Reference [35] derived the β-function of Yang-Mills theory by calculating the effective action of strings in a background magnetic field. Reference [36] pointed out a correspondence between loop amplitudes in the background field gauge and loops calculated using string motivated rules. More recently, [37, 38, 39] calculated two, three and four point gauge boson amplitudes in open bosonic string theory. Using a suitable prescription for continuing the amplitudes off-shell, the renormalization constants obtained from the zero slope limit of the string theory amplitudes were observed to respect the background field gauge Ward identities. We will see below that the low energy limit of the string theory amplitudes in our D-brane construction reproduce the field theory loop amplitudes calculated in the background field gauge.

In figure 2 we show the one loop diagrams for the two-point function in U(1) non-commutative gauge theory. In ordinary gauge theory the tadpole diagrams give vanishing contributions because

$$\int \frac{d^d p}{p^2} = 0 \tag{2.12}$$

in dimensional regularization. However, in the non-commutative theory these graphs are non vanishing and must be included to obtain a gauge invariant answer. The sum of the diagrams of figure 1 is

$$\begin{aligned} \Pi_{\mu\nu} = 2g^2 \int \frac{d^d q}{(2\pi)^d} \sin^2 \left(\frac{\tilde{p}q}{2} \right) \Bigg[& \frac{8(p^2 G_{\mu\nu} - p_\mu p_\nu)}{q^2 (p+q)^2} + \\ & + (d-2) \left(\frac{(p+2q)_\mu (p+2q)_\nu}{q^2 (p+q)^2} - \frac{2G_{\mu\nu}}{q^2} \right) \Bigg] , \end{aligned} \tag{2.13}$$

where $\tilde{p}^\mu = \theta^{\mu\nu} p_\nu$. Using the identity $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ the field theory expression separates into two parts, one independent of θ and one with a $\cos(\tilde{p}q)$ in

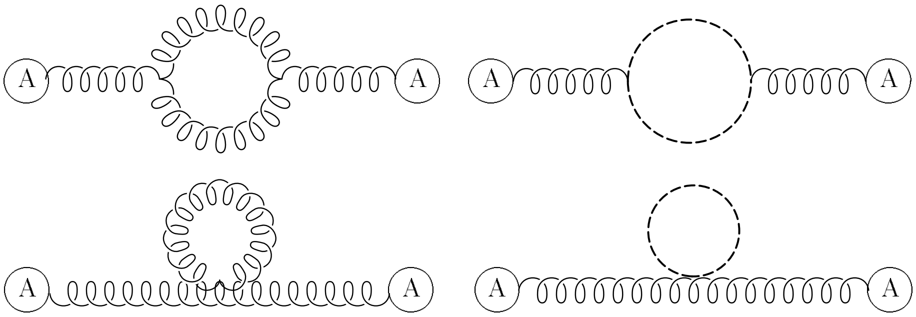


Figure 2: One loop contributions to the two-point function.

the integrand. The term independent of θ corresponds to the planar diagrams [10], and gives an expression identical to ordinary Yang-Mills theory with the usual group theory factor $f_{acd}f_{bcd} = N\delta_{ab}$ replaced by 2. This piece is divergent in four dimensions and gives a $1/\epsilon$ pole. From this term one can extract the β-function of the non-commutative theory [21, 14, 13]. The term with the $\cos(\tilde{p}q)$ corresponds to the non-planar graphs and is ultraviolet finite.

To compare with the string theory calculation in section 3 it is best to combine the propagators using Feynman parameters and then do the momentum integral via the method of Schwinger parameters. The contribution of the planar graphs is

$$\Pi^P_{\mu\nu} = \frac{ig^2\mu^{4-d}}{(4\pi)^{d/2}}(p^2 G_{\mu\nu} - p_\mu p_\nu)\int_0^\infty dt \int_0^1 dx\, t^{1-d/2} e^{-p^2 tx(1-x)} \times \\ \times \left(8 - (d-2)(1-2x)^2\right). \qquad (2.14)$$

The contribution from the non-planar graphs is

$$\Pi^{NP}_{\mu\nu} = -\frac{ig^2}{(4\pi)^2}\int_0^\infty dt \int_0^1 dx\, t^{-1} e^{-p^2 tx(1-x) - \tilde{p}^2/4t} \times \qquad (2.15) \\ \times \left[(p^2 G_{\mu\nu} - p_\mu p_\nu)(8 - 2(1-2x)^2) - \frac{2}{t^2}\tilde{p}_\mu \tilde{p}_\nu\right],$$

where since these diagrams are finite we have set $d = 4$.

The planar diagrams are ultraviolet divergent and to regulate this divergence we take $d = 4 - 2\epsilon$. The result of evaluating the integral in (2.14) is

$$\begin{aligned} \Pi^P_{\mu\nu} &= \frac{i2g^2}{(4\pi)^2}(p^2 G_{\mu\nu} - p_\mu p_\nu)\left(\frac{11 - 7\epsilon}{3 - 2\epsilon}\right)\left(\frac{-p^2}{4\pi\mu^2}\right)^{-\epsilon}\frac{\Gamma[\epsilon]\Gamma[1-\epsilon]^2}{\Gamma[2-2\epsilon]} \\ &= \frac{ig^2}{(4\pi)^2}\frac{22}{3}(p^2 G_{\mu\nu} - p_\mu p_\nu)\left(\frac{1}{\epsilon} - \ln\left(\frac{-p^2}{\mu^2}\right) + \cdots\right), \end{aligned} \qquad (2.16)$$

where $\cdots$ is a constant.

The non-planar diagrams evaluate to

$$\begin{aligned}\Pi^{NP}_{\mu\nu} &= -\frac{ig^2}{(4\pi)^2}\int_0^1 dx\,\Big[2\left(p^2G_{\mu\nu}-p_\mu p_\nu\right)\left(8-2(1-2x)^2\right)K_0\left(p\tilde{p}\sqrt{x(1-x)}\right)-\\ &\quad -16\frac{\tilde{p}_\mu\tilde{p}_\nu}{\tilde{p}^4}p^2\tilde{p}^2x(1-x)K_2\left(p\tilde{p}\sqrt{x(1-x)}\right)\Big]\\ &= -\frac{ig^2}{(4\pi)^2}\left[\left(p^2G_{\mu\nu}-p_\mu p_\nu\right)\left(-\frac{22}{3}\ln\left(p^2\tilde{p}^2\right)+\cdots\right)-32\frac{\tilde{p}_\mu\tilde{p}_\nu}{\tilde{p}^4}+\cdots\right],\end{aligned} \tag{2.17}$$

where we have expanded in $p^2\tilde{p}^2$ to lowest order and kept the most infrared singular terms. The non-planar graphs give rise to $\ln(p^2\tilde{p}^2)$, as first observed in [26], as well as the correction to the photon polarization tensor of the form $\tilde{p}_\mu\tilde{p}_\nu/\tilde{p}^4$, observed in [28, 30]. This last term is interesting since it modifies the photon dispersion relation.

We will see in the next section that the zero slope limit of the one loop string theory amplitude exactly reproduces the field theory answer (2.14) and (2.15). The Schwinger parameter t in the field theory calculation is proportional to the modulus of the string world sheet while the Feynman parameter x is related to the separation of the vertex operators on the worldsheet.

3. The string theory calculation

In this section we reproduce the field theory results of section 2 using string theory. We will find a simple realization of the non-commutative U(1) gauge theory using D3-branes in bosonic string theory and perform a one loop string calculation which will yield, in the massless open string boundary of moduli space, the results of the previous section.

A four-dimensional pure U(1) gauge theory can be realized by taking the low energy limit of the theory on a single D3-brane of bosonic string theory stuck at an $\mathbb{R}^{22}/\mathbb{Z}_2$ orbifold singularity. This can be accomplished [31, 32] by choosing the action of the orbifold group on the Chan-Paton factors to be represented by either of the two one-dimensional representations of $\mathbb{Z}_2$. The projection equation projects out the transverse scalars and we are left with only a gauge field. The non-commutative version of the pure U(1) gauge theory can be obtained by applying a constant magnetic field along the D-brane worldvolume.

The quantum effective action of this gauge theory is encoded in the string theory effective action. An appropriate truncation of a string loop diagram should provide, in the low energy limit, the field theory answer. We will explicitly verify at the one loop level that the planar and non-planar two-point amplitudes for the photon on the cylinder in a background magnetic field reproduce the corresponding computations in the non-commutative gauge theory in the background field gauge.

The correlation function of the photon field vertex operators on the disk shows that the low energy classical action on the brane is a non-commutative gauge theory with the usual replacement of conventional products by $\star$-products [9]. Comparison between the string theory calculation and the field theory fixes the normalization of the photon vertex operator to be

$$V(z,k) = ig\int_{\partial\Sigma} ds\ e\cdot\partial_s X e^{ik\cdot X}\,, \tag{3.1}$$

where g is the tree level Yang-Mills coupling constant, e_μ is the polarization of the photon, s is the coordinate on the worldsheet boundary $\partial\Sigma$ and indices are contracted with the $G^{\mu\nu}$ metric.

The worldsheet topology of the one-loop diagram is a cylinder, which we represent in the complex z-plane as a rectangle of width π and height $2\pi it$ — where $0\le t\le\infty$ is the modulus of the cylinder — and with the edges $y=0$ and $y=2\pi it$ identified:

$$\begin{aligned} &z = x+iy\,, \quad 0\le x\le\pi\,,\\ &y\simeq y+2\pi t\,, \quad 0\le y\le 2\pi t\,. \end{aligned} \tag{3.2}$$

Open string vertex operators must be inserted on the boundaries of the cylinder at either $x=0$ or $x=\pi$, with the positions given by $w=iy$ and $w=\pi+iy$, respectively.

The full two-point function is obtained by summing over the planar (two vertex operators on the same boundary) and non-planar (each vertex operator on a different boundary) diagrams. These diagrams are given by

$$A = \int_0^\infty \frac{dt}{2t}\int_0^{2\pi t} dy_1\int_0^{2\pi t} dy_2\ Z(t)\langle V(y_1,k_1)V(y_2,k_2)\rangle \equiv e_1^\mu e_2^\nu \Pi_{\mu\nu}\,, \tag{3.3}$$

where one should keep in mind when computing $\langle\cdots\rangle$ if the diagram is planar or non-planar. The different terms in (3.3) are easily understood. The $1/2t$ factor arises from explicitly gauge fixing the path integral and dividing by the conformal Killing volume of the cylinder (which allows all vertex operators to be unfixed). $Z(t)$ is the open string partition function of the vacuum under consideration, which in our case is that of an open string ending on D3-brane of bosonic string theory stuck at an orbifold singularity and in a background magnetic field. The correlator $\langle\cdots\rangle$ is computed by contracting the fields using the Green's function on the cylinder with boundary conditions modified by the background magnetic field.

The open string partition function is a key ingredient in the measure of the correlation function (3.3). Worldsheet consistency conditions require projecting the open string spectrum onto states invariant under the action of the orbifold group. For our $\mathbb{Z}_2$ orbifold this is reflected in the partition function

$$Z(t) = \mathrm{Tr}\left(\frac{1+g}{2}e^{-2tH_o}\right) = Z_1(t) + Z_g(t)\,, \tag{3.4}$$

where H_o is the open string hamiltonian and g is the $\mathbb{Z}_2$ generator. The $\mathbb{Z}_2$ action on the endpoints of the string corresponding to a stuck D3-brane just multiplies (3.4) by unity [31, 32, 40]. It is straightforward to show that

$$
\begin{aligned}
Z_1(t) &= i\det(g+2\pi\alpha' F)\frac{V_{p+1}}{2}(8\pi^2\alpha' t)^{-\frac{p+1}{2}}\eta(it)^{-24}\,,\\
Z_g(t) &= i\det(g+2\pi\alpha' F)2^{\frac{25-p}{2}}\frac{V_{p+1}}{2}(8\pi^2\alpha' t)^{-\frac{p+1}{2}}\vartheta_2(0,it)^{\frac{p-25}{2}}\eta(it)^{\frac{27-3p}{2}}\,,
\end{aligned}
\tag{3.5}
$$

where we have left explicit the dimensionality of the brane (which will turn out to be useful when comparing to the field theory results from section 2). $F_{\mu\nu}$ are the components of the background magnetic field.

In order to compute the correlation function (3.3) we must solve for the Green's function of the worldsheet scalars on the cylinder. The background magnetic field along the brane does not modify the equations of motion of the open strings ending on it, but it does change the boundary conditions on the worldsheet fields. Worldsheet coordinates along the brane satisfy the following boundary conditions:[2]

$$
g_{\mu\nu}\partial_n X^\nu + 2\pi i\alpha' F_{\mu\nu}\partial_s X^\nu\Big|_{\partial\Sigma} = 0\,. \tag{3.6}
$$

Here $g_{\mu\nu}$ is the closed string metric (the metric that appears in the string sigma model action). The operators ∂_n and ∂_s are derivatives normal and tangential to the worldsheet boundaries $\partial\Sigma$. The correlation function of vertex operators $\langle\cdots\rangle$ on a given worldsheet is computed from the propagators of the worldsheet fields, which can be found by solving the worldsheet wave equation while taking into account the boundary conditions (3.6). On the cylinder, the wave equation to be solved is

$$
\frac{2}{\alpha'}\partial_w\partial_{\bar w}G^{\rho\sigma}(w,w') = -2\pi\delta^2(w-w')g^{\rho\sigma} + \frac{1}{2\pi t}g^{\rho\sigma}\,. \tag{3.7}
$$

The last term, which is proportional to the inverse area of the cylinder, is included in order to satisfy Gauss' law and is compatible with the boundary conditions (3.6). The propagator we are interested in should solve (3.7), satisfy (3.6) at both boundaries of the cylinder, and respect the identification $y \simeq y + 2\pi t$ of the cylinder. The solution is given by

$$
\begin{aligned}
\mathcal{G}^{\mu\nu}(w,w') &\equiv \langle X^\mu(w)X^\nu(w')\rangle\\
&= -\alpha'\left[g^{\mu\nu}\left(\log\left|\vartheta_1\left(\frac{w-w'}{2\pi i t}|\frac{i}{t}\right)\right| - \log\left|\vartheta_1\left(\frac{w+\bar w'}{2\pi i t}|\frac{i}{t}\right)\right| - \right.\right.\\
&\qquad\left. - \frac{\mathrm{Re}^2(w+\bar w') + \mathrm{Re}^2(w-w')}{4\pi t}\right) + \\
&\qquad \left. + G^{\mu\nu}\log\left|\vartheta_1\left(\frac{w+\bar w'}{2\pi i t}|\frac{i}{t}\right)\right|^2 + \frac{\theta^{\mu\nu}}{2\pi\alpha'}\log\left(-\frac{\vartheta_1\left(\frac{w+\bar w'}{2\pi i t}|\frac{i}{t}\right)}{\vartheta_1^*\left(\frac{w+\bar w'}{2\pi i t}|\frac{i}{t}\right)}\right)\right]\,,
\end{aligned}
\tag{3.8}
$$

where w and w' are points on the cylinder.

[2] The coordinates transverse to the brane are projected out by the orbifold quotient.

The propagator on the cylinder has a similar structure to the propagator on the disk. Here $G^{\mu\nu}$ is the open string metric (the metric that defines the dispersion relation for open string fields), and $\theta^{\mu\nu}$ is the noncommutativity parameter which appears in the definition of the $\star$-product. They are defined by

$$
\begin{aligned}
G^{\mu\nu} &= \left(\frac{1}{g+2\pi\alpha' F} g \frac{1}{g-2\pi\alpha' F}\right)^{\mu\nu}, \\
\theta^{\mu\nu} &= -(2\pi\alpha')^2 \left(\frac{1}{g+2\pi\alpha' F} F \frac{1}{g-2\pi\alpha' F}\right)^{\mu\nu}.
\end{aligned} \tag{3.9}
$$

The non-commutative field theory is obtained by taking the limit $\alpha' \sim \epsilon \to 0$ and $g \sim \epsilon^{1/2} \to 0$, with the magnetic field kept constant [9].

For the two-point function of photon vertex operators we only need the propagator for points on the boundaries. The correlator is given by

$$
\begin{aligned}
&\langle \partial_s X^\mu e^{ik\cdot X}(w) \partial_s X^\nu e^{-ik\cdot X}(w')\rangle = \\
&\quad = \left(\partial_w \partial_{w'} \mathcal{G}^{\mu\nu} + k_\rho k_\sigma \partial_w \mathcal{G}^{\mu\rho} \partial_{w'} \mathcal{G}^{\sigma\nu}\right) e^{k_\rho k_\sigma \left(\mathcal{G}^{\rho\sigma}(w,w') - \frac{1}{2}\mathcal{G}_r^{\rho\sigma}(w,w) - \frac{1}{2}\mathcal{G}_r^{\rho\sigma}(w',w')\right)} \qquad (3.10)\\
&\quad = \left(-k_\rho k_\sigma \partial_w \mathcal{G}^{\rho\sigma} \partial_{w'} \mathcal{G}^{\mu\nu} + k_\rho k_\sigma \partial_w \mathcal{G}^{\mu\rho} \partial_{w'} \mathcal{G}^{\nu\sigma}\right) e^{k_\rho k_\sigma \left(\mathcal{G}^{\rho\sigma}(w,w') - \frac{1}{2}\mathcal{G}_r^{\rho\sigma}(w,w) - \frac{1}{2}\mathcal{G}_r^{\rho\sigma}(w',w')\right)},
\end{aligned}
$$

where we have integrated the first term by parts. $\mathcal{G}_r^{mn}(w,w)$ is the renormalized propagator, which regulates the divergences in the self-contractions by subtracting the short distance behaviour of the propagator. The proper renormalized propagator for open string vertex operators is given by

$$
\mathcal{G}_r^{\rho\sigma}(w,w') = \mathcal{G}^{\rho\sigma}(w,w') + \alpha' G^{\rho\sigma}(\log|w-w'|^2), \tag{3.11}
$$

where w and w' are points on the same boundary. We will denote the combination in the exponent in (3.10) as

$$
\widetilde{\mathcal{G}}^{\rho\sigma}(w,w') \equiv \mathcal{G}^{\rho\sigma}(w,w') - \frac{1}{2}\mathcal{G}_r^{\rho\sigma}(w,w) - \frac{1}{2}\mathcal{G}_r^{\rho\sigma}(w',w'). \tag{3.12}
$$

We will now consider in turn the results for the planar and non-planar diagrams.

3.1 Planar two-point function

$\widetilde{\mathcal{G}}^{\rho\sigma}(w,w')$ differs from $\mathcal{G}^{\rho\sigma}(w,w')$ by a term that is independent of the position of the vertex operators. On the $x=0$ boundary it is given by[3]

$$
\begin{aligned}
\widetilde{\mathcal{G}}^{\rho\sigma}(w,w') &= -\alpha' \left[G^{\rho\sigma} \log\left| 2\pi \frac{\vartheta_1\left(\frac{i(y-y')}{2\pi}, it\right)}{\vartheta_1'(0,it)}\right|^2 - \frac{(y-y')^2}{2\pi t}\right] + \frac{i}{2}\theta^{\rho\sigma}\epsilon(y-y') \\
&\equiv -\alpha' G^{\rho\sigma}\Gamma(y-y') + \frac{i}{2}\theta^{\rho\sigma}\epsilon(y-y'),
\end{aligned} \tag{3.13}
$$

[3] Unlike eq.(3.8) above, this expression is not manifestly periodic. However, it is straightforward to rewrite this expression in a form in which periodicity is manifest.

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where $w = y, w' = y'$ (or $w = \pi + iy, w' = \pi + iy'$), $G^{\rho\sigma}$ is the open string metric and $\epsilon(x)$ is 1 for $x > 0$ and -1 for $x < 0$. On the $x = \pi$ boundary the sign of the term proportional to $\theta^{\rho\sigma}$ changes sign.[4] Note that we have used a ϑ-function identity to rewrite the propagator in a form conducive to taking the $t \to \infty$ limit.

Plugging this expression into the correlator (3.10) we see that it has the familiar form of the vacuum polarization diagram of the photon

$$\langle \partial_s X^\mu e^{ik\cdot X}(w_1)\partial_s X^\nu e^{-ik\cdot X}(w_2)\rangle = -\alpha'^2 \left(k^2 G^{\mu\nu} - k^\mu k^\nu\right)(\partial_y \Gamma)^2 e^{-\alpha' k^2 \Gamma}\,, \tag{3.14}$$

where $k^2 = G^{\rho\sigma}k_\rho k_\sigma$ and $y = y_1 - y_2$.

Combining all the terms in (3.3) one is led to the following expression for the planar two-point function

$$\Pi^P_{\mu\nu} = -g^2 \int_0^\infty \frac{dt}{2t} \int_0^{2\pi t} dy_1 \int_0^{2\pi t} dy_2\ Z(t)\ \alpha'^2 \left(k^2 G_{\mu\nu} - k_\mu k_\nu\right)(\partial_y\Gamma)^2 e^{-\alpha' k^2 \Gamma}\,, \tag{3.15}$$

where $Z(t)$ is given by (3.5) and Γ by (3.13).

The task at hand is to identify in this string computation the non-commutative field theory result. We have to examine (3.15) in the decoupling limit specified in [9], which in particular requires taking the $\alpha' \to 0$ limit. In this limit we only get contributions from corners of string moduli space. We will now show that we obtain the exact planar field theory answer from the boundary of moduli space of the cylinder which is dominated by massless open strings,[5] which comes from the $t \to \infty$ limit. We therefore need the large t expression of the integrand in (3.15). The large t expansions of $Z(t)$ and of $\partial_y\Gamma$ are given by

$$\begin{aligned} Z(t) &\simeq V_{p+1}(8\pi^2\alpha' t)^{-\frac{p+1}{2}}\left(e^{2\pi t} + p - 1 + \mathcal{O}(e^{-2\pi t})\right)\,, \\ \partial_y \Gamma &\simeq 1 - 2x + 2\left(e^{-2\pi x t} - e^{2\pi x t}e^{-2\pi t}\right)\,, \end{aligned} \tag{3.16}$$

where $x = y/2\pi t$. Plugging these expressions into (3.15) and tossing out the contribution due to the tachyon, one gets (with $d = p + 1$ and $g \to g\mu^{4-d}$)

$$\begin{aligned} \Pi^P_{\mu\nu} = i\frac{g^2\mu^{4-d}}{(4\pi)^{d/2}}\left(k^2 G_{\mu\nu} - k_\mu k_\nu\right) \int_0^\infty dt \int_0^1 dx\ t^{1-d/2} \times \\ \times \left(8 - (d-2)(1-2x)^2\right) e^{-k^2 t x(1-x)}\,, \end{aligned} \tag{3.17}$$

which is precisely the field theory answer (2.14). To obtain this result we rescaled $t \to t/\alpha'$ and $y \to y/\alpha'$. In these new variables (with $\alpha' \to 0$) any finite value of t is in the extreme open string limit of the moduli space. In particular the excited open string corrections in (3.16) become $\mathcal{O}(e^{-2\pi t/\alpha'})$ which vanish in the decoupling limit. This will be discussed further in section 4.

[4] We thank H. Dorn for correspondence on this point.

[5] Note that since we are using bosonic string theory the field theory result is obtained only after removing by hand the divergence caused by the open string tachyon.

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3.2 Non-planar two-point function

The non-planar propagator is given by

$$\widetilde{\mathcal{G}}^{\rho\sigma} = -\alpha' \left[G^{\rho\sigma} \left(\log \left| 2\pi \frac{\vartheta_2\left(\frac{i(y-y')}{2\pi}, it\right)}{\vartheta_1'(0,it)} \right|^2 + \frac{\pi}{2t} - \frac{(y-y')^2}{2\pi t} \right) \pm \right.$$
$$\left. \pm\, i \frac{\theta^{\rho\sigma}}{2\pi\alpha'} \frac{(y+y')}{t} - g^{\rho\sigma} \frac{\pi}{2t} \right]. \tag{3.18}$$

The sign of the the term proportional to θ in (3.18) depends on which boundary the vertex operators lie. If $w = iy$ and $w' = \pi + iy'$ then the sign should be taken to be positive, while it is negative if $w = \pi + iy$ and $w' = iy'$. As in the planar diagram computation we expand (3.18) in the large t region. The asymptotics of the term proportional to the open string metric $G^{\rho\sigma}$ is the same as for the planar diagram. The important differences are in the terms proportional to the closed string metric $g^{\mu\nu}$ and the noncommutativity parameter $\theta^{\mu\nu}$. The term proportional to $\theta^{\mu\nu}$ does not contribute to the exponential in (3.10), but it plays a very important role in the derivative terms.

Since this is a non-planar diagram for an oriented string, there is an overall factor of -1 since the ends of the string carry opposite charges. The final answer is given by

$$\Pi^{NP}_{\mu\nu} = -i \frac{g^2 \mu^{4-d}}{(4\pi)^{d/2}} \int_0^\infty dt \int_0^1 dx\, t^{1-d/2} e^{-k^2 t x(1-x) - \tilde{k}^2/4t} \times \tag{3.19}$$
$$\times \left(\left(k^2 G_{\mu\nu} - k_\mu k_\nu\right) \left(8 - (d-2)(1-2x)^2\right) - \frac{2}{t^2} \tilde{k}^\mu \tilde{k}^\nu \right).$$

This expression is identical to that obtained from the non-planar field theory graphs (2.15).

4. Discussion

In the previous sections we have seen how the annulus (cylinder) amplitude of string theory in a background F field reproduces, in the decoupling limit of [9], the planar and non-planar results of non-commutative gauge theory. Let us discuss this in more detail. Schematically the two-point function on the annulus is given in the open string channel by

$$A \sim \int_0^\infty dt\; t \sum_I \exp\left(-\Delta_I t\right). \tag{4.1}$$

The index I labels all open string states and Δ_I, basically the L_0 eigenvalue, is the mass squared of state I plus momentum dependence. In the decoupling limit of [9] $\alpha' \to 0$ and hence the string mass scale is sent to infinity. The oscillator contribution to Δ_I is unaffected by F and hence is just N_I/α' where N_I is the total oscillator occupation number. So in the $\alpha' \to 0$ limit all the excited open string states become much heavier than the massless one and hence should decouple from the dynamics. The vanishing of the exponential corrections in (3.17) and (3.19) illustrates this. This is similar to other decoupling limits such as those which show that field theories arise from branes separated by short distances. As pointed out in [26] and further discussed in [27, 30] there are peculiar singularities indicating IR/UV mixing in the non-planar non-commutative gauge theory results. These results have been interpreted to mean [26, 27] that some closed string residue remains in the field theory, even in the decoupling limit. So it is important to examine if and how decoupling is breaking down here.

Generally, the only way decoupling can fail is for the interactions of the decoupled theory to have bad high energy behavior. If loops of massless open string states are UV divergent, then massive open string states will generally be excited, violating decoupling. If the decoupled field theory is divergent but renormalizable then there will be a mild violation of decoupling, but all the effects of the massive string states can be absorbed in a few "renormalized" couplings. For instance, $g^2 \ln(-p^2/\mu^2)$ in (2.16) becomes, in string theory, $g^2(\ln(-p^2\alpha') + \mathcal{O}(1))$.

Now let us examine decoupling in the non-planar diagrams that display mysterious UV/IR singularities. We can write a caricature of the string amplitude (3.19) by suppressing the x integral, all numerical factors, and adding back in the effect of the first excited open string state.

$$A_{NP} \sim \int dt\; t^{1-d/2} e^{-k^2 t - \tilde{k}^2/4t}(1 + e^{-2\pi t/\alpha'} + \cdots)\,. \tag{4.2}$$

Note that the $\tilde{k}^2/4t$ term in the exponential renders the UV region of the modular integration ($t \to 0$) completely finite for any non-zero $\tilde{k}^2$. This term is present in the field theoretic amplitude (2.15) and represents the effect of the rapidly oscillating phases in the non-commutative gauge theory interaction vertices. These phases are enough to render the non-planar amplitude finite for any non-zero $\tilde{k}^2$.

The smallest important value of t in (4.2) is roughly $t \sim \tilde{k}^2$. The contribution of the first excited state is then $\sim e^{-2\pi\tilde{k}^2/\alpha'}$. In the decoupling limit $\alpha' \to 0$ and this contribution vanishes for any non-zero $\tilde{k}^2$. The decoupled field theory amplitude is UV finite so decoupling cannot fail.

To further investigate this question let us keep α' finite. There are then two regimes to consider. If $\tilde{k}^2 \gg \alpha'$ then the excited open string state contribution is negligible and the decoupled field theory result is accurate. If $\tilde{k}^2 \ll \alpha'$, however, the small t region of the integral may be important. This depends on whether the field theory graph without phase factors is UV divergent. It will be, for instance, if the space-time dimension d is large enough. If there is a small t UV divergence then all the excited open string states will become important. In this case the correct way to analyze the situation is to use channel duality and rewrite the amplitude in terms of closed string states. At small t only the lightest closed string states will contribute, giving a massless propagator $1/k^2$ (assuming we drop the closed string tachyon).

The region where the closed string description is valid becomes smaller and smaller as we approach the decoupling limit $\alpha' \to 0$. In this limit the region of validity shrinks to a set of measure zero. For all finite $\tilde{k}^2$ the decoupled field theory describing only the lightest open string mode is exact. So the complete structure of the mysterious IR/UV singularities is contained in the open string description.

Of course one can formally represent the behavior of the lightest open string in the dual closed string channel. But this requires a sum over closed string states of arbitrarily high mass and does not seem very transparent. This is the usual situation in dualities. A regime that has a simple description in one set of variables typically has a complicated description in the dual variables.

To illustrate this point consider (4.2) for general d. The log divergence in $d = 4$ becomes a $1/\tilde{k}^{d-4}$ divergence. The massless closed strings will produce a $1/k^2$ behavior for any d. To produce the open string behavior will require a sum over all the closed string states.

There is at least one situation where decoupled field theory results can be reproduced from the lightest closed string state.[6] This is the case where a non-renormalization theorem exists [41, 42, 43]. The contribution of excited open string states vanishes, typically because they are in long multiplets of an extended supersymmetry algebra. The exact amplitude is given by the lightest open string state, and so this must also agree with the closed string answer. This mechanism seems to require lots of supersymmetry, and usually applies only to special amplitudes. So it seems questionable whether it will be helpful in giving a general explanation for these mysterious singularities.

Note added. In the past week the papers [44, 45, 46] appeared on the archive. They overlap substantially with ours.

Acknowledgments

We would like to thank Allan Adams, Sergei Gukov, John McGreevy, Hirosi Ooguri, Maxim Perelstein, John Schwarz, Lenny Susskind, Nick Toumbas and Edward Witten for discussions. We would like to thank Iain Stewart for help making the figures. J.G. and T.M. are supported in part by the DOE under grant no. DE-FG03-92-ER 40701. M.R. is supported by the Caltech Discovery Fund under grant no. RFBR 98-02-16575 and DE-FG-05-ER 40219. M.K. is supported by an NSF graduate fellowship. S.S. is partially supported by NSF grant 9870115.

[6]This observation is due to Lenny Susskind.

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